

# On The Use of Empirical Bayes for Comparative Interrupted Time Series with an Application to Mandatory Helmet Legislation

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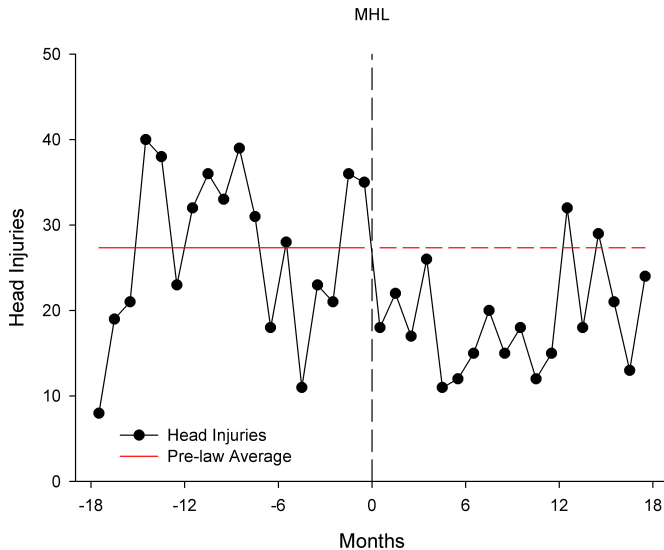
# Outline

- 1 Motivating Example
- 2 Interrupted Time Series
- 3 Empirical Bayes ITS
- 4 Results
- 5 Discussion

# Mandatory bicycle helmet legislation in NSW

- Intervention directed at increasing helmet wearing among cyclists
  - → Lower bicycle related head injuries
  - Not a panacea for all bicycle related injuries
- Applies to all age groups
- Went into effect in two stages
  - Adults (>16): 1 January 1991
  - Children: 1 July 1991
- Led to greater helmet wearing rates (~25% to ~80%)
- Associated with fewer bicycle related head injuries

# Adult head injury hospitalisations in NSW



# Criticisms of MHL

- MHL is very controversial
- Leads to reductions in cycling?
  - Fewer cyclists → fewer bicycle related head injuries?
- Leads to increased risk to cyclists
  - Risk compensation, rotational injuries, safety in numbers?
- Has a negative health economics impact?
  - Quit cycling → no exercise → more obesity?
  - Morbidity/mortality from obesity outweighs safety benefit of helmets?
- Loss of freedom?
- Debate rages on after 20+ years
- The anti-helmet advocacy group Bicycle Helmet **Research** Foundation is the main proponent of these criticisms<sup>1</sup>

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<sup>1</sup>[www.cyclehelmets.org](http://www.cyclehelmets.org)

# BIG QUESTIONS

## Question 1

Is the drop in head injury associated **solely, partly or not at all** with the helmet law?

## Question 2

Did the **helmet law CAUSE** the drop in head injury? (via increased helmet wearing)

## Question 3

Did **declines in cycling CAUSE** the drop in head injury?


# Causal Inference for Population-based Interventions

- Pre- and post-intervention periods are not randomised

⇒ **Causal inference is difficult**

- Relevant data is often missing
  - cycling exposure, risk of injury
- Routinely collected data is probably best option for assessment
  - hospitalisation data, census data, police data (traffic, criminal reports)
- A rigorous analysis is paramount
  - There are many examples where different analyses result in different conclusions<sup>2</sup>
- What is the **best** analytic method/framework?
  - **Interrupted time series** is most common

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<sup>2</sup>Ramsay et al. (2003) "Interrupted time series designs in health technology assessment: Lessons learned from two systematic reviews of behavior change strategies." 

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# Interrupted Time Series (ITS)

- Type of quasi-experimental design
  - Participants are not randomised
- Estimates a time series before and after an intervention
  - Comparing single pre- and post-intervention effects can hide *history*
  - Multiple pre- and post-intervention observations avoids *regression to the mean*
- Important comparisons made between pre- and post-intervention time series
  - Change in level (immediate impact)
  - Change in slope (gradual impact)

## Interrupted time series (basic structural model)

$$y_t = \mu_t + \gamma_t + \sum_{j=1}^k \delta_j x_{jt} + \lambda w_t + \varepsilon_t$$

$\mu_t$  := trend

$\gamma_t$  := seasonal component

$x_{jt}$  := jth explanatory variable

$\delta_j$  := coefficient for  $x_{jt}$

$\lambda$  := intervention effect

$w_t$  := pre/post-law indicator

$\varepsilon_t$  := irregular component

## Effects are additive

- Outcome is comprised of

$$\left( \begin{array}{c} \text{basic} \\ \text{pattern} \end{array} \right) + \left( \begin{array}{c} \text{cyclical} \\ \text{effects} \end{array} \right) + \left( \begin{array}{c} \text{other} \\ \text{effects} \end{array} \right) + \left( \begin{array}{c} \text{law} \\ \text{effects} \end{array} \right) + \left( \begin{array}{c} \text{random} \\ \text{noise} \end{array} \right)$$

$$\log(y_T) = \beta_0 + \beta_1 T + \beta_2 I + \beta_3 TI + u_T$$

where

$T :=$  time

$I := \begin{cases} 0 & \text{pre-intervention} \\ 1 & \text{post-intervention} \end{cases}$

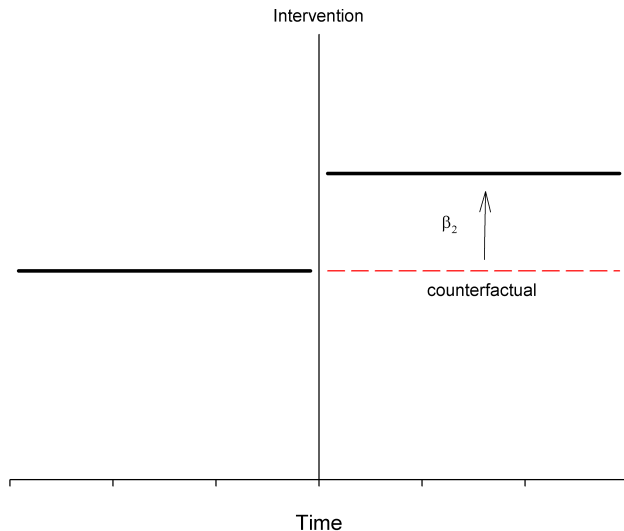
$u_T :=$  error process (time dependent?)

- Could also include cyclical effects or other (confounding) variables
- **Counterfactual** (or trajectory) is the estimated time series if the intervention had not occurred, for example

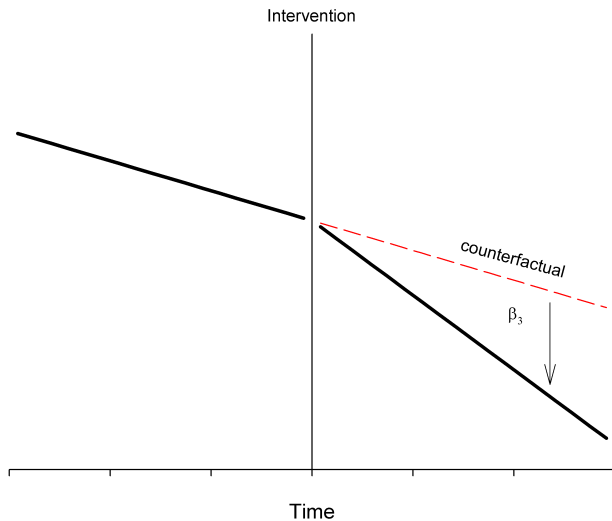
$$\log(\hat{y}_T) = \hat{\beta}_0 + \hat{\beta}_1 T$$

- $\beta_2$  and  $\beta_3$  are comparisons between the counterfactual and the post-intervention model

# Change in Level



# Change in Slope



# Threat to Internal Validity

- Unmeasured confounding is a major weakness of ITS
- The use of a control/comparator time series is often recommended<sup>3</sup>
  - Also affected by unmeasured confounding
  - Not subject to the intervention
  - Observations over the same time period
  - Could be a related observation from the same study unit
- Treatment and control are modelled simultaneously
  - Comparative interrupted time series (CITS)

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<sup>3</sup>Shadish, Cook & Campbell (2002) *Experimental and Quasi-experimental designs for generalized causal inference*.

$$\log(y_T) = \beta_0 + \beta_1 T + \beta_2 I + \beta_3 C + \beta_4 TI + \beta_5 TC + \beta_6 IC + \beta_7 TIC + u_T$$

where

$$C := \begin{cases} 1 & \text{primary time series} \\ 0 & \text{comparative time series} \end{cases}$$

$u_T$  := error process (time dependent?)

- The comparison of the two time series is

$$\log(y_T^p / y_T^c) = (\beta_3 + \beta_6 I) + (\beta_4 + \beta_7 I) T$$

- $\beta_6$  and  $\beta_7$  are comparisons between the counterfactual and the post-intervention model **relative to the comparative time series**
- Assumes unmeasured confounding factors are identical for  $y_T^p$  and  $y_T^c$  and therefore cancel out



# How to Choose a Comparative Time Series?

## Question 4

How do we know whether a comparative time series has accounted for unmeasured confounding?

## Question 5

Given multiple comparators, how do you choose the best one?

# How to Choose a Comparative Time Series?

- 1 Linden and Adams (2011) recommend choosing a comparative time series that is *similar* to the primary time series **before** the intervention<sup>4</sup>
  - Only time varying component?
- 2 Walter et al. (2013) chose comparative time series based on highest within-time period correlation<sup>5</sup>
  - What if unmeasured confounders are not similar?

$$\phi = \frac{\text{cov}(\varepsilon_t^P, \varepsilon_t^C)}{\sqrt{\text{var}(\varepsilon_t^P)\text{var}(\varepsilon_t^C)}} \neq \frac{\text{cov}(\eta_t^P + \varepsilon_t^P, \eta_t^C + \varepsilon_t^C)}{\sqrt{\text{var}(\eta_t^P + \varepsilon_t^P)\text{var}(\eta_t^C + \varepsilon_t^C)}}$$

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<sup>4</sup>Linden & Adams (2011) "Applying a propensity score-based weighting model to interrupted time series data: improving causal inference in programme evaluation"

<sup>5</sup>Walter, Olivier, Churches & Grzebieta (2013) "The impact of compulsory helmet legislation on cyclist head injuries in New South Wales, Australia: A response"

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- Basic idea:
  - 1 Pre-intervention data is used to estimate a *prior* model
  - 2 This model is extrapolated over the post-intervention period (i.e., counterfactual)
  - 3 Post-intervention observations are analysed relative to the counterfactual (*posterior*)

- Pre-intervention model

$$E\left(\log(y_T^{EB})\right) = \alpha_0 + \alpha_1 T + \alpha_2 C + \alpha_3 TC, \quad T < 0$$

- Counterfactual residuals

$$\Delta_T = \log(y_T) - \log(\hat{y}_T^{EB}), \quad T > 0$$

- No intervention effect when  $\bar{\Delta}_T = 0$
- Residuals will have systematic pattern if unmeasured confounders are not similar

- Including a comparative time series

$$\Delta_T^p - \Delta_T^c = \log(y_T^p / y_T^c) - \log(\hat{y}_T^{EB-p} / \hat{y}_T^{EB-c})$$

- No **relative** intervention effect when  $\bar{\Delta}_T^p - \bar{\Delta}_T^c = 0$
- Residuals will have systematic pattern if unmeasured confounders are not similar

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- Hospital presentations from 1 July 1989 to 30 June 1992
- Cases identified from ICD-9-CM
- Primary outcome: bicycle-related head injury hospitalisations
- Possible comparators
  - Bicycle-related arm injury hospitalisations (no head injury)
  - Bicycle-related leg injury hospitalisations (no head injury)
  - Pedestrian-related head injury hospitalisations
  - Australian beer production (sensitivity analysis?)

# 1st and 2nd Criteria

- Results from CITS models for each comparator

Comparator	Pre-law similarity $\hat{\beta}_5$ (SE)	Within-time correlation $\hat{\phi}$
Arm	-0.008 (0.015)	0.026
Leg	0.023 (0.021)	0.096
Head-Peds	-0.008 (0.020)	-0.063
Beer	<b>0.003 (0.015)</b>	<b>0.185</b>

- Australian beer production is the “best” comparator using these criteria



# Empirical Bayes Criterion

- Models were fit to pre-intervention data using each potential comparator
- Linear models fit to counterfactual residuals

Comparator	Intercept	Slope
Arm	-0.263 (0.138)	0.010 (0.013)
Leg	-0.263 (0.157)	-0.025 (0.015)
Head-Peds	-0.383 (0.190)	0.001 (0.018)
Beer	-0.494 (0.165)	0.010 (0.016)

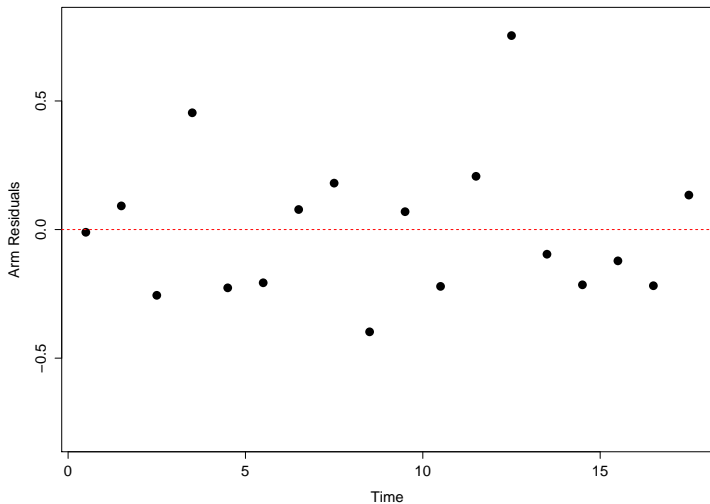
- All slope estimates are statistically non-significant and “small”
- All intercept estimates are statistically significant (or nearly so)

- Head injuries had the greatest relative decline compared to Australian beer production

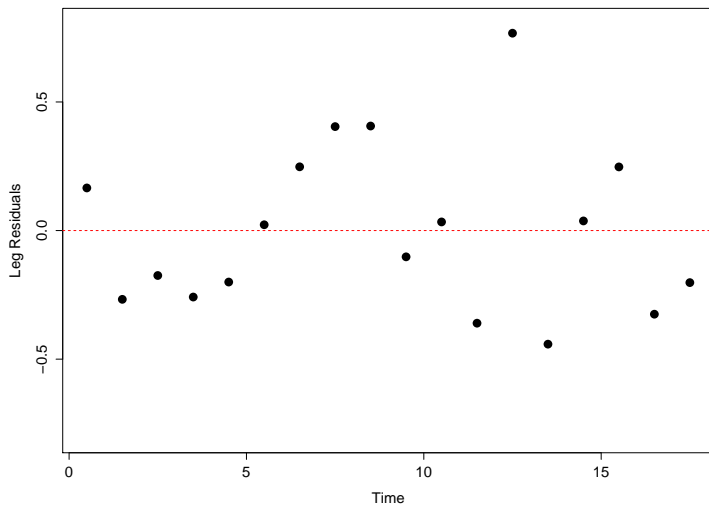
$$\exp(-0.494) - 1 = -39\%$$

- Is Australian beer production the “best” comparator to cycling head injury hospitalisations?
- Residual analysis suggests cycling arm injuries are affected by similar unmeasured confounding

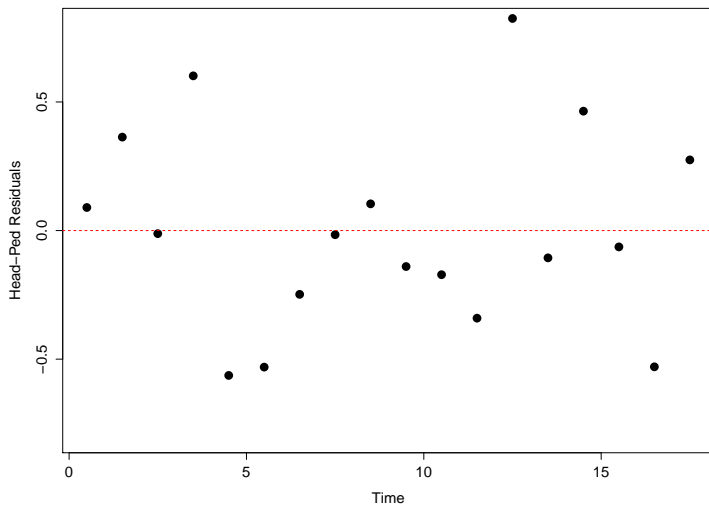
# Arm Residuals



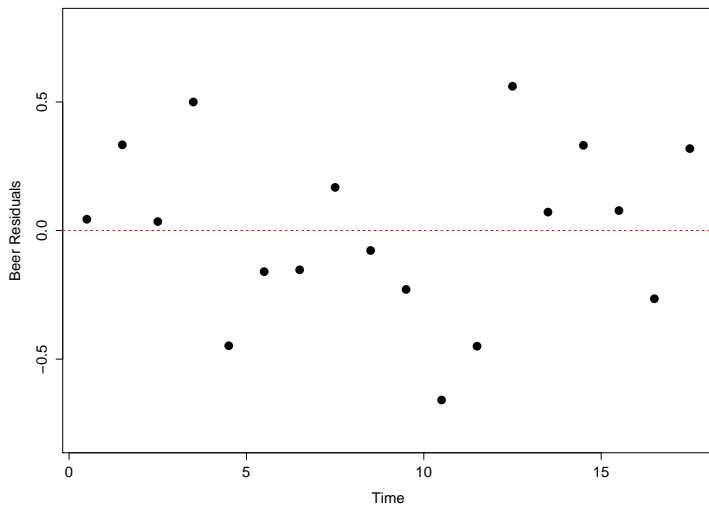
# Leg Residuals



# Pedestrian-Head Residuals



# Beer Residuals



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# Which Comparator is Best?

- Linden and Adams criterion
  - All do not differ significantly in pre-law period (Beer production better than others)
- Walter et al. criterion
  - Beer production exhibits largest within-month correlation
- Empirical Bayes (residual analysis) criterion
  - Arm injury residuals appear random
  - Systematic pattern for others → invalid statistical inference?
- Estimated intervention effect is smallest relative to arm injuries
  - Most conservative estimate



- Causal inference for population-based interventions is difficult
- Interrupted time series is likely the best analytic approach
  - Threats to internal validity (due to lack of randomisation)
- The use of a comparative time series is promising
  - An analytic framework for choosing “best” comparator is needed

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Thank You!

Questions?